Method of Orienting Lines for Minimizing a Sum of Euclidean Norms

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$$\min_t \left(\mathcal{F}(t_1, \ldots, t_k) = \sum_{i=1}^{k-1} \|p_i(t_i) - p_{i+1}(t_{i+1})\| \right), \qquad (1)$$

where $t_i \geq a_i > 0$, $p_i : \boldsymbol{R}^1 \to \boldsymbol{R}^2$, $(i = 1, \ldots, k)$ are linear, $k \geq 3$ and $\|\cdot\|$ is an Euclidean norm in $\boldsymbol{R}^2 \Longrightarrow$

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- How can we find exact solutions of Problem $(1)/(1^*)$?

Geometrical Form of Problem (1)

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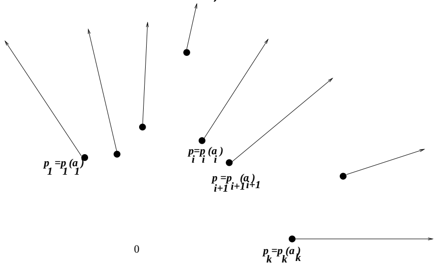

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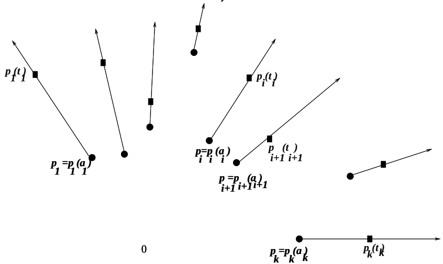

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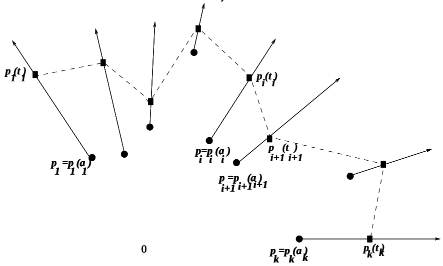

Optimal Control Problem

Some optimal control problems can be stated in the form

$$(P) \qquad \min_{x,u} \int_{t_0}^{t_f} F(t, x(t), u(t))dt$$

subject to

$$\dot{x}(t) = f(t, x(t), u(t)), \ \ t \in [t_0, t_f]$$

$$\alpha_1(t) \geq x(t) \geq \alpha_2(t), x(t_0) = x_0, x(t_f) = x_f$$

$$g(t, u(t)) \geq 0, \ \ t \in [t_0, t_f].$$

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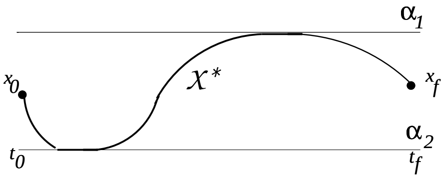
Geometrical form of (P):



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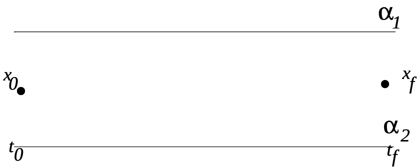

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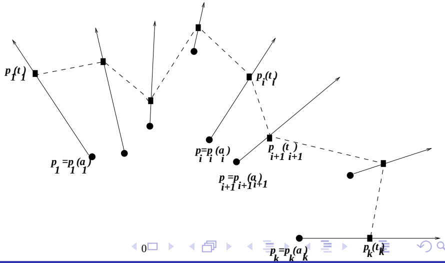

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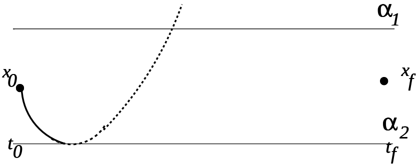

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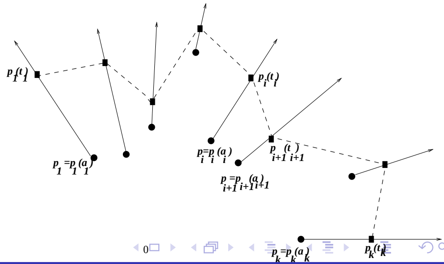

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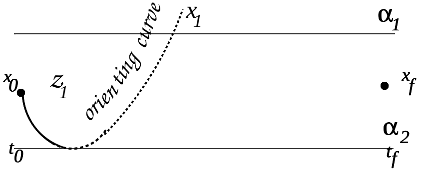

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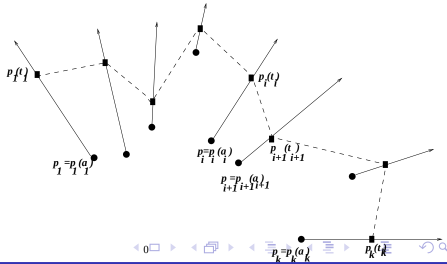

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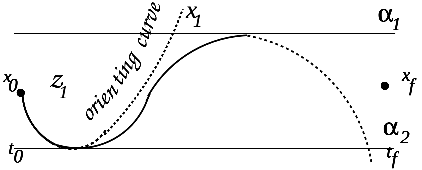

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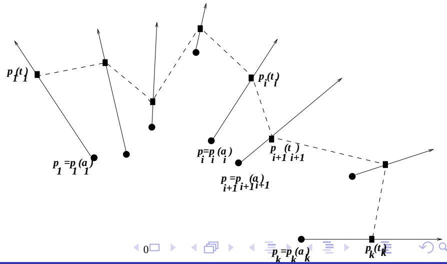

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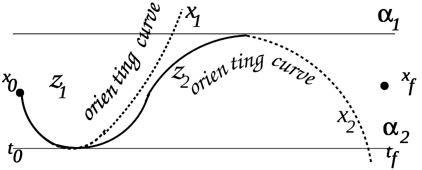

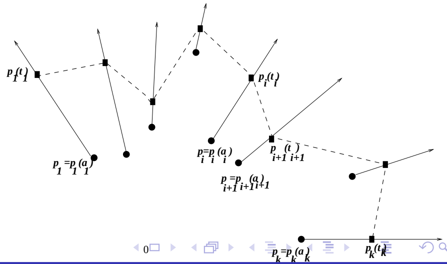

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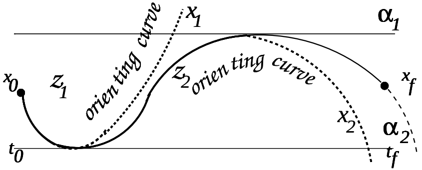

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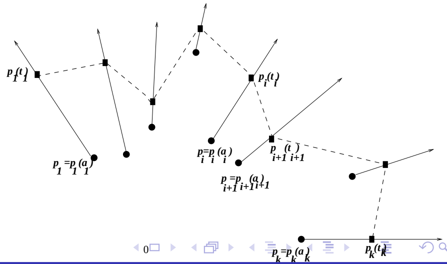

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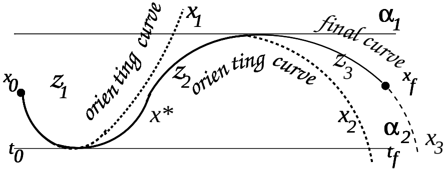

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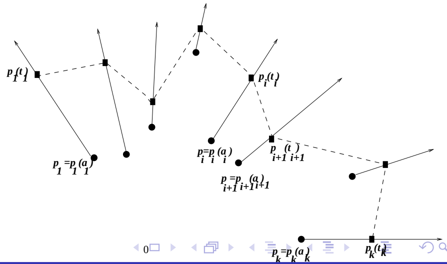

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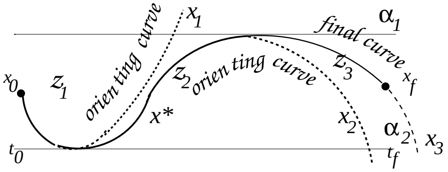
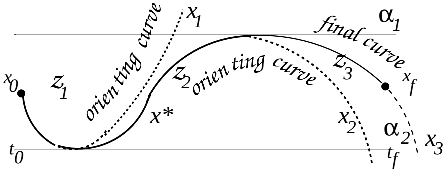
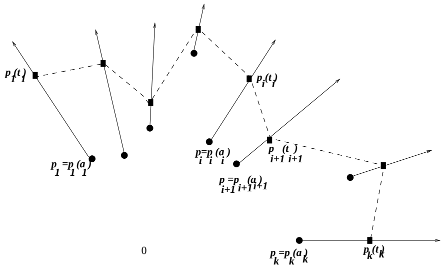


Method of Orienting Curves

• Optimal control problem (P):
Final curve
Orienting curve
optimal solution consists of parts of orienting curves and a final curve.

• Minimizing a Sum of Euclidean Norms (1): Can it be solved exactly by the idea of the Method of Orienting Curves above?
Final curve → Final line? Orienting curve → Orienting line?
Difficulty: First and final points, boundaries α_1, α_2 are <u>unknown</u>!!!





Method of Orienting Lines

Take $\|p_m(a_m)\| = \max_{1\leq i\leq k}\{\|p_i(a_i)\|\}$. Then $p_m(a_m)$ belongs to the solution $Q(a)$ of Problem (1).
Let α be

the sector of the circle

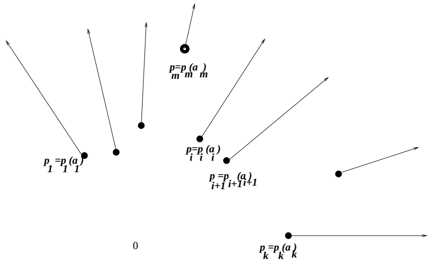
radius $\max\{\|p_i\| : m \leq i \leq k\}$ centered at 0,

between two rays $\overrightarrow{0p_m}$ and $\overrightarrow{0p_k}$ contains $p_m(a_m), \ldots, p_i(a_k)$.

$\Longrightarrow$ The path formed by the solution $Q(a)$ of Problem (1) is the shortest path inside the domain formed by the polyline $\alpha_1 := p_m(a_m) \ldots p_i(a_k)$ and $\alpha_1 := \text{arc}_\alpha$ with unknown final point.

Method of Orienting Lines

→ Boundaries:
$\alpha_1 := p_m(a_m) \ldots p_i(a_k)$, $\alpha_1 := \text{arc}_\alpha$.
→ We start from $p_m(a_m)$ to construct final line and orienting lines



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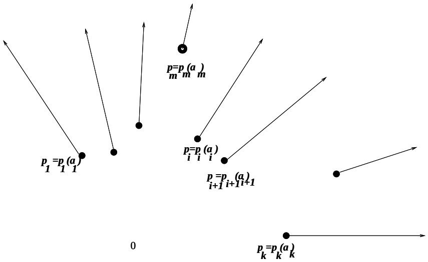

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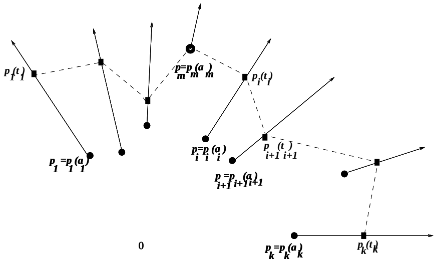

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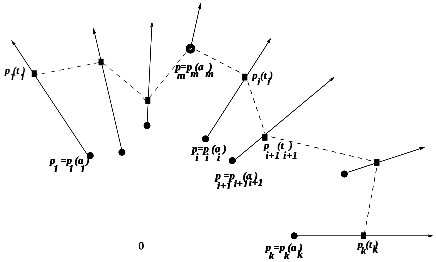

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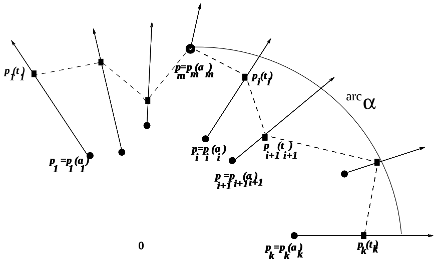

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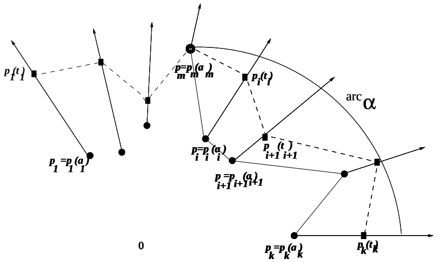

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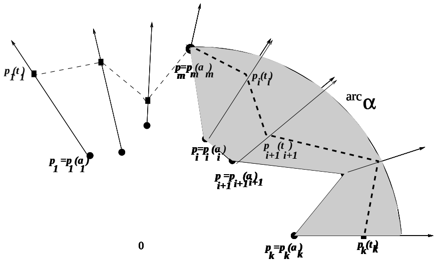


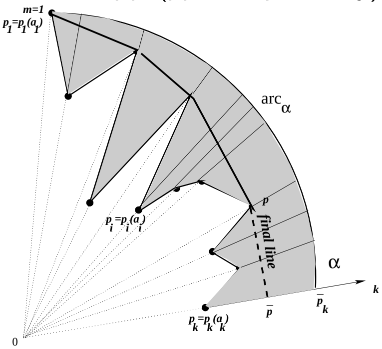
Final Line and Orienting Line

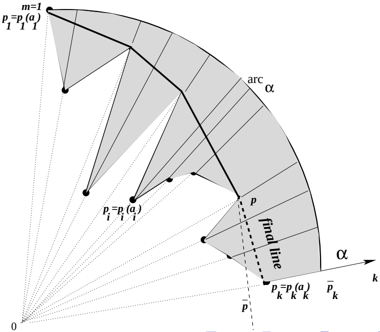
Let $p \in Q(a)$ (solution of Problem (1)), $\bar{p} \in 0p_k$ such that $p\bar{p} \perp \overrightarrow{0p_k}$

z is on or right of $p\bar{p}$ $\forall z \in Q(a) \setminus \{p_k\}$, z between p and p_{k-1}

p_k is right (on or left, respectively) of $p\bar{p}$

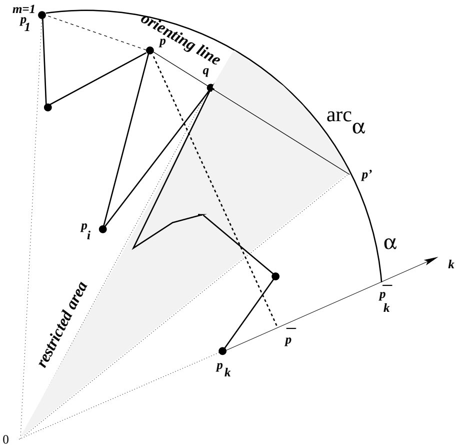
$\Longrightarrow$ $p\bar{p}$ (pp_k, respectively) is a *final line* through p.





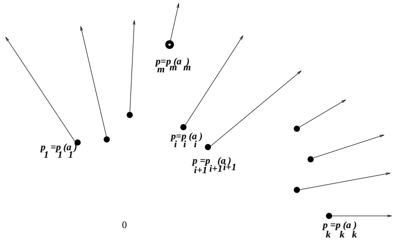
Final Line and Orienting Line

$p, q \in Q(a) \setminus \{p_k\}$ such that q between p and p_{k-1}.
z is on or right of $\overrightarrow{pq}$ $\forall z \in Q(a)$ between p and p_k,
$\Longrightarrow$ pq is an *orienting line* through p.



Exact Algorithm

We now determine $Q(a) = Q^R(t^*)$ (a part of solution of Problem (1)).



Exact solution of (1) consists of parts of orienting lines and final lines.

1. Begin at p_m. Set $l = 1$. Then, $q_1 = p_m$.

2. Consider q_l.

If there is a final line through q_l go to **4**

else, there is an orienting line through q_l, go to **3**.

3. Let $q_l p_{k_l^*}$ be an orienting line with $p_{k_l^*}$ as its transfer point. Then, $p_{k_l^*} \in Q^R(t^*)$. Set $q_{l+1} = p_{k_l^*}$ and $l = l + 1$, go to **2**.

4. Let $q_l q$ be the final line, where $q \in \overrightarrow{p_k}$. Then $Q^R(t^*)$ includes $\{q_1, \ldots, q_l, q\}$.

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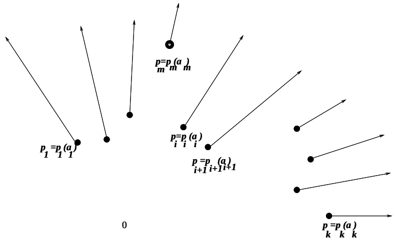

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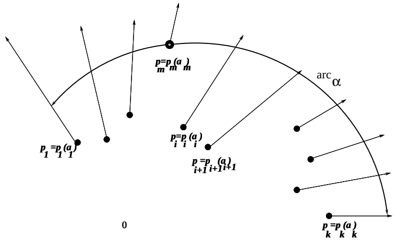

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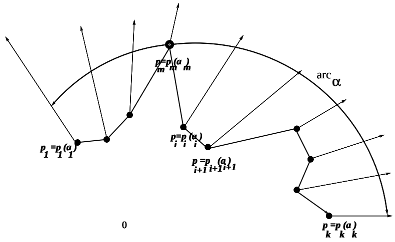

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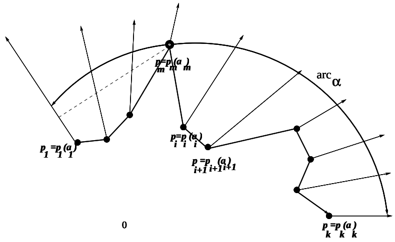

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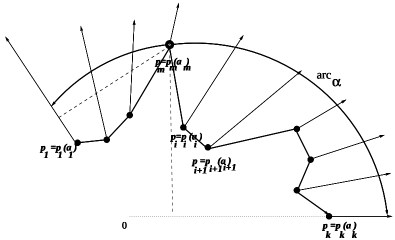

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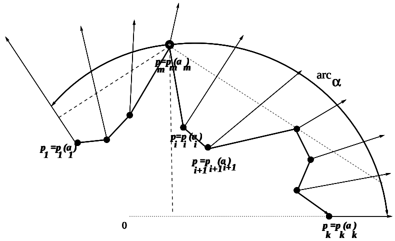

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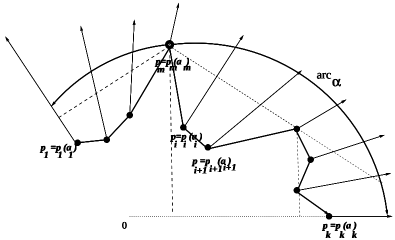

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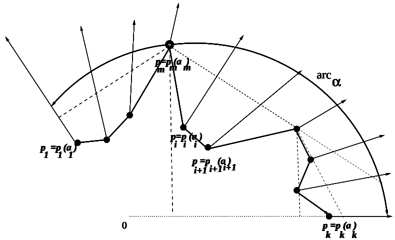

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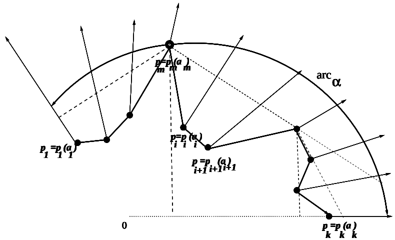

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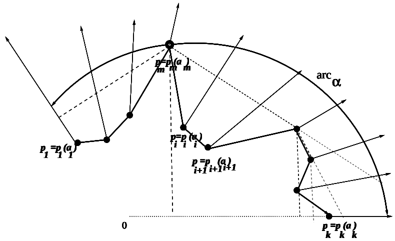

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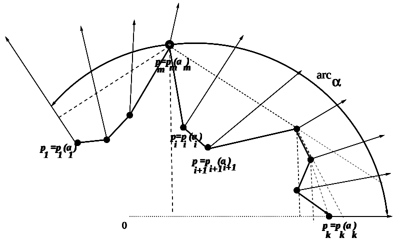

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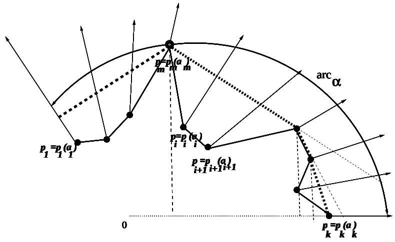

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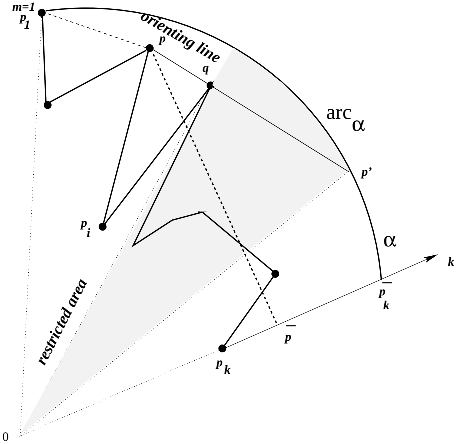
Restricted Areas

It helps to determine quickly if $q_l p$ is orienting line or not:



Application Example

$$\inf_t \mathcal{F}(t) \tag{2}$$

where

$$\begin{aligned}\mathcal{F}(t) \;=\; & \sqrt{(2t_7 - 30t_6)^2 + 36t_6^2} + \sqrt{(30t_6 - 53t_5)^2 + (6t_6 - 16t_5)^2} \\ & + \sqrt{(53t_5 - 4t_4)^2 + (16t_5 - 5t_4)^2} + \sqrt{(4t_4 - 3t_3)^2 + (5t_4 - 7t_3)^2} \\ & + \sqrt{(3t_3 - t_2)^2 + (7t_3 - 21t_2)^2} + \sqrt{(-5t_1 - t_2)^2 + (5t_1 - 21t_2)^2}\end{aligned}$$

and $t_7 \geq a_7 = 1$, $t_6 \geq a_6 = 1.5$, $t_5 \geq a_5 = 1$, $t_4 \geq a_4 = 14$, $t_3 \geq a_3 = 3$, $t_2 \geq a_2 = 5$ and $t_1 \geq a_1 = 3.5$.

Application Example

	Approximation algorithms	Our exact algorithm
	–(2) written as a second order cone program –interior-point methods	–Final lines, orienting lines –Restricted area
$\mathcal{F}(t^*)$	≈ 209.636414	$= 56/5 + 1512/265 + 2814/53 + \left(\sqrt{646594} + \sqrt{1530400}\right)/33 + 55\sqrt{2}$
t_1^*	≈ 9.99998	$= 10$
t_2^*	≈ 5	$= 5$
t_3^*	≈ 11.9829	$= 395/33$
t_4^*	≈ 14	$= 14$
t_5^*	≈ 1.0566	$= 56/53$
t_6^*	≈ 1.8666	$= 28/15$
t_7^*	≈ 28.0002	$= 28$

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  This method does not rely on triangulation and graph tools.
- *Open question:* Can minimizing a sum of Euclidean norms in higher dimmensions with nonlinear p_i be solved *exactly* in the same manner? $\Longrightarrow$ Exact solution consists of parts of orienting Xs and final Xs???

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