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"When Irreducibles are Prime"
(joint work Davide Rinaldi)

Abstract:

Many an indirect proof with Zorn's Lemma has recently allowed
for being turned upside down into a direct proof with Raoult's
Open Induction. Under sufficiently concrete circumstances this
may even yield a constructive proof without any form of the
Axiom of Choice. To prepare the ground for a more systematic
treatment we now classify the cases that can be found in
mathematical practice by way of representative proof patterns. To
start with, we distill a generalised form of the contrapositive of
the Separation Lemma or Prime Ideal Theorem that is ascribed to
Lindenbaum, Krull, Stone, and Tarski. Our version subsumes not
only instances from different branches of algebra but also a
Henkin-style completeness proof for first-order logic. By
recurrence to a theorem of McCoy, Fuchs, and Schmidt on
irreducible ideals we further shed light on why prime ideals occur
--- and why transfinite methods.