Monique Laurent CWI "Semidefinite programming hierarchies for combinatorial problems via lift-and-project and sums of squares of polynomials"

Abstract:

A fundamental problem in combinatorial optimization is how to optimize a linear function over the set of 0/1-valued points lying in some given set K; geometrically, how to find the convex hull P of these integral points. Lift-and-project is a widely used paradigm permitting to generate valid linear inequalities for the polytope P in a systematic way. Various methods exist, whose applicability depends on the given presentation of the set K. For instance, K could be a convex set given by a separation oracle, or a basic closed semi-algebraic set given by polynomial equations and inequalities.

A common feature of these methods is that they generate a hierarchy of (linear or semidefinite) relaxations for the polytope P, converging to it in finitely many steps.

Key ingredients include using sums of squares representations for positive polynomials and the dual theory of moments.